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Wave propagation in a general anisotropic poroelastic medium with anisotropic permeability: phase velocity and attenuation

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Abstract

Wave propagation is studied to characterize an oil reservoir provided reservoir rocks could be considered a general anisotropic poroelastic solid saturated by a viscous fluid with flow controlled by the anisotropic permeability of the porous solid. Biot's theory is used to derive a modified Christoffel equation for the propagation of plane harmonic waves in such a medium. This equation is solved further to get a biquadratic equation whose roots represent the complex velocities of four attenuating quasi-waves in such a medium. These complex velocities define the phase velocities of propagation and quality factors of attenuation of all the quasi-waves propagating along a given phase direction in three-dimensional space. The variations of phase velocities and attenuation factors with the direction of phase propagation are computed, for a realistic numerical model. Propagation regimes for anisotropic/isotropic poroelastic media with isotropic/anisotropic permeability are obtained as reduced cases.

Keywords: Anisotropic poroelasticity; Anisotropic permeability; Phase velocity; Attenuation; Reservoir

1. Introduction

The study of mechanical behaviour of porous media is of special importance in the seismic exploration, for the closer description of physical phenomena around the oil reservoirs. A reservoir is, no doubt, a fluid-saturated porous solid medium pervaded by aligned cracks. In the presence of aligned cracks, an elastic medium behaves anisotropic to wave propagation (Crampin, 1981). In general, the seismic anisotropy is caused by the lithological and crystal alignments, stress induced effects, aligned cracks and fluid-filled pores. The absence of point symmetry of pores may cause the anisotropy of arbitrary symmetry. Inherent cracks (particularly microcracks) and pores in a crystal rock are modified and aligned by the changes in stress-field of the rock and pore fluid pressure. The distribution of stress-aligned fluid-filled microcracks and

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preferentially oriented pore space is known as extensive dilatancy anisotropy (EDA) and has been recognised (Crampin, 1994) throughout much of the crust. Crampin (1987) discussed the geological and industrial implications of EDA. In exploration studies (Helbig, 1984; Leary and Henyey, 1985; Kerner et al., 1989; Corrigan, 1989; and many others), the velocity anisotropy measured from travel times has suggested the presence of significant anisotropy in sedimentary basins.

The porosity and permeability are two fundamental parameters which are economically important in oil production. Whereas, porosity is the most important geometrical property, the permeability is an equally important physical property of a porous medium. Permeability measures the ability of a porous medium to conduct fluid flow in its pores. Reservoir rocks can exhibit a strong permeability anisotropy. There may or may not be some correlation between elastic anisotropy and permeability (or hydraulic) anisotropy (Rasolofosaon and Zinszner, 2002). To infer hydraulic transport properties of reservoir rocks from seismic data is a difficult but important aspect of exploration studies.

This paper provides a mathematical model to study the phase propagation velocities and attenuations of four quasi-waves in reservoir rocks. Their variations with phase direction are studied numerically.

2. Anisotropic poroelasticity

Following Biot (1955, 1956), the governing equations for a fluid-saturated porous media, in the absence of body forces, are

$$\begin{aligned} \sigma_{ij,j} &= \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i + b_{ij}(\dot{u}_j - \dot{U}_j), \\ \sigma_{,i} &= \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i - b_{ij}(\dot{u}_j - \dot{U}_j). \end{aligned} \tag{1}$$

In these equations, u_i and U_i are the components of the average displacements for the solid and fluid phases, respectively. The dot notation is used to represent differentiation with respect to time. Indices can take the values 1–3. Summation convention is valid for repeated indices. The comma (,) before an index represents partial space differentiation. ρ_{11}, ρ_{12} and ρ_{22} are the dynamical constants depending upon the porosity of solid, fluid–solid coupling and densities of solid particles and interstitial fluid. Flow-resistance symmetric matrix $\{b_{ij}\}$ steers the effects of frequency (ω), fluid viscosity (μ), solid–matrix permeability (χ) and porosity (f) on the wave propagation. Following Biot (1956),

$$\mathbf{b} = \frac{\mu}{\chi_0} f^2 \{\chi_a^{-1}\}, \tag{2}$$

where, χ_0 is norm of permeability tensor. χ_a is a symmetric matrix of order 3 and represents normalized anisotropic permeability tensor. This expression of $\mathbf{b}$ is valid, only, for the low-frequency range, where the flow in the pores is of Poiseuille type. For higher frequencies, a correction factor is applied to the viscosity, μ, replacing it by $\mu F(\kappa)$. With $\bar{a}$ denoting the linear dimension of pores, the $\kappa = \bar{a}\sqrt{\omega\rho_f/\mu}$. $F(\kappa)$, a complex function of frequency ω, is, then, defined as follows:

When $\kappa \ll 1$ (i.e., highly viscous fluid and/or smaller pores),

$$F(\kappa) = [1 + \kappa^4/1152 + \mathrm{o}(\kappa^6)] + \imath[\kappa^2(1 - \kappa^4/1440)/24 + \mathrm{o}(\kappa^8)]$$

and, when $\kappa \gg 1$ (i.e., low-viscosity fluid and/or wider pores),

$$F(\kappa) = \frac{\kappa}{4\sqrt{2}}\left[\left(1 + \frac{3\sqrt{2}}{2\kappa} - \frac{15}{8\kappa^2} - \frac{135}{128\kappa^4}\right) + \imath\left(1 - \frac{15}{8\kappa^2} - \frac{15\sqrt{2}}{8\kappa^3} - \frac{135}{128\kappa^4}\right)\right].$$

For an anisotropic porous material, the constitutive equations for stresses in the solid phase (i.e. σ_{ij}) and fluid (i.e. σ) are

$$\sigma_{ij} = c_{ijkl}u_{k,l} + m_{ij}U_{k,k},$$
$$\sigma = m_{ij}u_{i,j} + RU_{k,k}. \tag{3}$$

The coefficients c_{ijkl}, m_{ij} and R are the material constants of a linear porous material. The assumptions $c_{ijkl} = c_{klij} = c_{jikl}, m_{ij} = m_{ji}$ and strain-energy considerations reduce the number of these material constants to be at the most 28, for general anisotropy.

To seek the harmonic solution of (1), for the propagation of plane waves, write

$$u_j = S_j \exp\{i\omega(p_k x_k - t)\},$$
$$U_j = F_j \exp\{i\omega(p_k x_k - t)\} \quad (j = 1, 2, 3), \tag{4}$$

where, ω is angular frequency and (p_1, p_2, p_3) is slowness vector. In terms of phase velocity v, the slowness is $\{p_1, p_2, p_3\} = \{n_1, n_2, n_3\}/v$, where n_j denotes the components of a unit vector normal to wave surface. Define row matrix $\mathbf{N} = (n_1, n_2, n_3)$ to represent the direction of phase propagation. Substituting (2) in (1) and, then, using (3), yields a system of six homogeneous equations in $S_1, S_2, S_3, F_1, F_2, F_3$. Non-trivial solution of this system of equations defines a modified Christoffel equation for the wave propagation in an anisotropic poroelastic medium. Eliminating F_j $(j = 1, 2, 3)$, the Christoffel equation is reduced to a system of three homogeneous equations, given by

$$W_{ij}S_j = 0 \quad (i = 1, 2, 3). \tag{5}$$

The W_{ij} are elements of a square matrix of order 3, which is defined as

$$\mathbf{W} = \left(\frac{\mathbf{X}_1}{d_0} - r_{11}\mathbf{I} - \mathbf{d}\right)h + \mathbf{Z} + \left(\mathbf{X}_2 - \mathbf{X}_1\frac{d_1}{d_0}\right)\frac{1}{d_0} + \frac{[\mathbf{X}_3 - (\mathbf{X}_2 - \mathbf{X}_1 d_1/d_0)d_1/d_0]h + \mathbf{X}_4}{h(d_0 h + d_1)}, \tag{6}$$

where, $\mathbf{I}$ is identity matrix of order 3 and dissipation matrix $\mathbf{d} = \imath\frac{\mathbf{b}}{\omega\rho_{22}}$. The variable $h = \rho_{22}v^2/R$ and $r_{1j} = \rho_{1j}/\rho_{22}$ $(j = 1, 2)$. Other variables, in (6), are expressed as follows:

$$d_0 = \det(\mathbf{d} + \mathbf{I}),$$

$$d_1 = d_{12}^2 n_3^2 + 2d_{12}(d_{33} + 1)n_1 n_2 - (d_{11} + 1)(d_{22} + 1)n_3^2 - 2d_{12}d_{23}n_1 n_3 + d_{13}^2 n_2^2 + 2d_{13}(d_{22} + 1)n_1 n_3 - (d_{11} + 1)(d_{33} + 1)n_2^2 - 2d_{13}d_{23}n_1 n_2 + d_{23}^2 n_1^2 + 2d_{23}(d_{11} + 1)n_2 n_3 - (d_{22} + 1)(d_{33} + 1)n_1^2 - 2d_{12}d_{13}n_2 n_3.$$

$$\mathbf{X}_1 = r_{12}^2\Phi - r_{12}(\mathbf{d}\Phi + \Phi\mathbf{d}) + \mathbf{d}\Phi\mathbf{d},$$
$$\mathbf{X}_2 = r_{12}^2\Gamma - r_{12}(\mathbf{d}\Gamma + \Gamma\mathbf{d} + \mathbf{E}'\mathbf{N}\Phi + \Phi\mathbf{N}'\mathbf{E}) + (\mathbf{d}\Gamma\mathbf{d} + \mathbf{E}'\mathbf{N}\Phi\mathbf{d} + \mathbf{d}\Phi\mathbf{N}'\mathbf{E}),$$
$$\mathbf{X}_3 = -r_{12}(\mathbf{E}'\mathbf{N}\Gamma + \Gamma\mathbf{N}'\mathbf{E}) + (\mathbf{d}\Gamma\mathbf{N}'\mathbf{E} + \mathbf{E}'\mathbf{N}\Gamma\mathbf{d} + \mathbf{E}'\mathbf{N}\Phi\mathbf{N}'\mathbf{E}),$$
$$\mathbf{X}_4 = \mathbf{E}'\mathbf{N}\Gamma\mathbf{N}'\mathbf{E}, \tag{7}$$

where, matrix $\Phi = \mathrm{adj}(\mathbf{d} + \mathbf{I})$ and elements of symmetric matrix Γ are defined as

$$\Gamma_{11} = 2d_{23}n_2 n_3 - (d_{22} + 1)n_3^2 - (d_{33} + 1)n_2^2 \quad \text{and} \quad \Gamma_{23} = d_{23}n_1^2 - (d_{11} + 1)n_2 n_3 - d_{21}n_3 n_1 - d_{31}n_2 n_1.$$

The other elements of Γ are obtained in cyclic order. The matrix $\mathbf{Z}$ in (6) is defined as follows:

Consider a general anisotropic poroelastic medium with elastic constants c_{ijkl} of the solid matrix represented by two-suffix notation, c_{ij}. Define, following Sharma (2002),

$$\alpha = \mathbf{N}\mathbf{A}_1\mathbf{N}', \quad \beta = \mathbf{N}\mathbf{A}_2\mathbf{N}', \quad \gamma = \mathbf{N}\mathbf{A}_3\mathbf{N}',$$
$$\delta = \mathbf{N}\mathbf{A}_4\mathbf{N}', \quad \eta = \mathbf{N}\mathbf{A}_5\mathbf{N}', \quad \zeta = \mathbf{N}\mathbf{A}_6\mathbf{N}', \tag{8}$$

where $\mathbf{N}'$ denotes the transpose of row matrix $\mathbf{N}$. $\mathbf{A}_1$–$\mathbf{A}_6$ are square matrices of order 3. For general anisotropy, these are defined as follows:

$$\begin{aligned} &\mathbf{A}_1 = \{a_{11}, a_{16}, a_{15}; a_{16}, a_{66}, a_{56}; a_{15}, a_{56}, a_{55}\}, \quad \mathbf{A}_2 = \{a_{66}, a_{26}, a_{46}; a_{26}, a_{22}, a_{24}; a_{46}, a_{24}, a_{44}\}, \\ &\mathbf{A}_3 = \{a_{55}, a_{45}, a_{35}; a_{45}, a_{44}, a_{34}; a_{35}, a_{34}, a_{33}\}, \quad \mathbf{A}_4 = \{a_{16}, a_{12}, a_{14}; a_{66}, a_{26}, a_{46}; a_{56}, a_{25}, a_{45}\}, \\ &\mathbf{A}_5 = \{a_{15}, a_{14}, a_{13}; a_{56}, a_{46}, a_{36}; a_{55}, a_{45}, a_{35}\}, \quad \mathbf{A}_6 = \{a_{56}, a_{46}, a_{36}; a_{25}, a_{24}, a_{23}; a_{45}, a_{44}, a_{34}\}, \end{aligned} \tag{9}$$

where $a_{ij} = c_{ij}/R$ and symmetric, square matrix

$$\mathbf{Z} = \{\alpha, \delta, \eta;\ \delta, \beta, \zeta;\ \eta, \zeta, \gamma\}. \tag{10}$$

The row matrix $\mathbf{E}$, in (7), is given by $E_i = \frac{1}{R} n_j m_{ij}$ ($i = 1, 2, 3$). The system of equation (5) is possible for all values of h other than 0 and $-d_1/d_0$.

3. Christoffel equation

For the non-trivial solution of the system of equation (5), the determinant of matrix W must vanish. This gives a biquadratic equation in h $(= \rho_{22} v^2/R)$, given by

$$h^4 - c_1 h^3 + c_2 h^2 - c_3 h + c_4 = 0. \tag{11}$$

The coefficients c's in (11) are complex and this implies that four roots of this equation may be complex. Therefore, the four waves propagating in such a medium are attenuating waves. The same directions of propagation and attenuation vectors of these waves, make them homogeneous waves. Let h_j ($j = 1, 2, 3, 4$), denotes the roots of this equation. The complex phase velocities of the four waves, given by $v_j = \sqrt{(Rh_j/\rho_{22})}$ ($j = 1, 2, 3, 4$), will be varying with the direction of phase propagation. These waves are called quasi-waves because polarizations may not be along the dynamic axes. Analogous to the propagation in an isotropic poroelastic medium (Section 4), these waves, represented by $j = 1, 2, 3, 4$, may be called the $qP1$-, $qP2$-, $qS1$- and $qS2$-waves, respectively. The complex velocity of a quasi-wave j, i.e., $v_j = v_R + \iota v_I$, defines the phase propagation velocity $V_j = (v_R^2 + v_I^2)/v_R$ and attenuation quality factor $Q_j^{-1} = -2v_I/v_R$ for the corresponding wave. The coefficients in Eq. (11) are expressed as follows:

$$\begin{aligned} &c_1 = -(d_0 T_2 + d_1 T_1)/(d_0 T_1), \quad c_2 = (T_5 + d_0 T_3 + d_1 T_2)/(d_0 T_1), \\ &c_3 = -(T_6 + d_0 T_4 + d_1 T_3)/(d_0 T_1), \quad c_4 = (T_7 + d_1 T_4)/(d_0 T_1), \end{aligned} \tag{12}$$

where

$$\begin{aligned} &T_1 = [\mathbf{AAA}], \quad T_2 = [\mathbf{AAB}] + [\mathbf{ABA}] + [\mathbf{BAA}], \quad T_3 = [\mathbf{ABB}] + [\mathbf{BAB}] + [\mathbf{BBA}], \quad T_4 = [\mathbf{BBB}], \\ &T_5 = [\mathbf{AAC}] + [\mathbf{ACA}] + [\mathbf{CAA}], \\ &T_6 = [\mathbf{AAD}] + [\mathbf{ABC}] + [\mathbf{ACB}] + [\mathbf{ADA}] + [\mathbf{BAC}] + [\mathbf{BCA}] + [\mathbf{CAB}] + [\mathbf{CBA}] + [\mathbf{DAA}], \\ &T_7 = [\mathbf{ABD}] + [\mathbf{ADB}] + [\mathbf{BAD}] + [\mathbf{BBC}] + [\mathbf{BCB}] + [\mathbf{BDA}] + [\mathbf{CBB}] + [\mathbf{DAB}] + [\mathbf{DBA}]. \end{aligned}$$

$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$, the square matrices of order 3, are defined by

$$\mathbf{A} = \frac{\mathbf{X}_1}{d_0} - r_{11}\mathbf{I} - \mathbf{d}, \quad \mathbf{B} = \left(\mathbf{X}_2 - \mathbf{X}_1 \frac{d_1}{d_0}\right)\frac{1}{d_0} + \mathbf{Z},$$

$$\mathbf{C} = \mathbf{X}_3 - \left(\mathbf{X}_2 - \mathbf{X}_1 \frac{d_1}{d_0}\right)\frac{d_1}{d_0}, \quad \mathbf{D} = \mathbf{X}_4.$$

The symbol $[\mathbf{ABC}]$ defines the determinant of a matrix $\{A_{11}, A_{12}, A_{13};\ B_{21}, B_{22}, B_{23};\ C_{31}, C_{32}, C_{33}\}$.

The four roots of Eq. (11) are written as

$$\begin{aligned} h_1 &= 0.5(-G-L+\Delta_1), \quad h_2 = 0.5(-G-L-\Delta_1), \\ h_3 &= 0.5(-G+L+\Delta_2), \quad h_4 = 0.5(-G+L-\Delta_2), \end{aligned} \tag{13}$$

where

$$\Delta_1 = \sqrt{(G+L)^2 - 4(H+M)}, \quad \Delta_2 = \sqrt{(G-L)^2 - 4(H-M)}, \quad G = -0.5c_1, \quad M = \sqrt{H^2 - c_4},$$
$$L = (0.5c_3 + GH)/M, \quad \left(L = \sqrt{G^2 - c_2 + 2H} \quad \text{if } M = 0\right)$$

and H is a root of the cubic equation

$$8H^3 - 4c_2H^2 + 2(c_1c_3 - 4c_4)H + c_4(4c_2 - c_1^2) - c_3^2 = 0.$$

4. Reduced cases

Isotropic permeability is represented by $d_{ij} = d_d\delta_{ij}$. It, further, yields $d_0 = (d_d+1)^3, d_1 = -(d_d+1)^2$, $\Phi = (d_d+1)^{-2}\mathbf{I}, \Gamma = (d_d+1)(\mathbf{N}'\mathbf{N} - \mathbf{I})$. Vanishing of $\mathbf{D}$ yields the reduced terms $T_6 = [\mathbf{ABC}] + [\mathbf{ACB}] + [\mathbf{BAC}] + [\mathbf{BCA}] + [\mathbf{CAB}] + [\mathbf{CBA}]$, and $T_7 = [\mathbf{BBC}] + [\mathbf{BCB}] + [\mathbf{CBB}]$. The elastic isotropy of porous solid is represented four elastic constants a_{11}, a_{66}, Q, R and $m_{ij} = Q\delta_{ij}$. The matrix $\mathbf{Z}$, given by (10), reduces to $\mathbf{Z} = a_{66}\mathbf{I} + (a_{11} - a_{66})\mathbf{N}'\mathbf{N}$. The corresponding changes in $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ modify the coefficients c_j and then the roots h_j.

The above given reductions enable to study the propagation regimes in the following media:

(i) anisotropic poroelastic medium with isotropic permeability,
(ii) isotropic poroelastic medium with anisotropic permeability,
(iii) isotropic poroelastic medium with isotropic permeability,
(iv) anisotropic poroelastic medium without dissipation (i.e., $\mathbf{d} = 0$),
(v) isotropic poroelastic medium without dissipation (i.e., $\mathbf{d} = 0$).

In addition to the above, the propagation regimes for various anisotropic symmetries (both elastic and hydraulic) can, also, be deduced. The reduced propagation regime for medium (iv) is verified with that in Sharma (2004). The propagation in medium (v) represents the standard Biot's theory and is verified as follows:

$$\mathbf{E} = (Q/R)\mathbf{N}; \quad \mathbf{Z} = a_{66}\mathbf{I} + (a_{11} - a_{66})\mathbf{N}'\mathbf{N}; \quad c_1 = (a_{11} + 2a_{66} + r_{11} - 2r_{12}Q/R)/g_0;$$
$$c_2 = (a_{11} - Q^2/R^2)/g_0 + a_{66}\{2(a_{11} + r_{11} - 2r_{12}Q/R) + a_{66}\}/g_0^2;$$
$$c_3 = 2a_{66}(a_{11} - Q^2/R^2)/g_0^2 + a_{66}^2(a_{11} + r_{11} - 2r_{12}Q/R)/g_0^3; \quad c_4 = a_{66}^2(a_{11} - Q^2/R^2)/g_0^3.$$

Using these relations, the biquadratic equation (10) reduces to

$$\{h^2 - (a_{11} + r_{11} - 2r_{12}Q/R)h/g_0 + (a_{11} - Q^2/R^2)/g_0\}(h - a_{66}/g_0)^2 = 0 \tag{14}$$

and its roots are given by

$$\begin{aligned} h_1 &= \left\{ (a_{11} + r_{11} - 2r_{12}Q/R) + \sqrt{(a_{11} + r_{11} - 2r_{12}Q/R)^2 - 4(a_{11} - Q^2/R^2} \right\} \Big/ (2g_0), \\ h_2 &= \left\{ (a_{11} + r_{11} - 2r_{12}Q/R) - \sqrt{(a_{11} + r_{11} - 2r_{12}Q/R)^2 - 4(a_{11} - Q^2/R^2} \right\} \Big/ (2g_0), \\ h_3 &= h_4 = a_{66}/g_0. \end{aligned} \tag{15}$$

Here, h_1, h_2 represent P_f, P_s waves, respectively, and identical roots h_3 and h_4 represent the only shear wave in isotropic medium, $g_0 = r_{11} - r_{12}^2$. The wave velocities $v_j = \sqrt{Rh_j/\rho_{22}}$ ($j = 1, 2, 3$), are the same as defined in Biot's theory.

Propagation regimes (i) and (iii) can also be obtained by generalizing the non-dissipative regimes (iv) and (v), respectively. It is done with the transformations of dynamical constants, given by

$$r_{11} \rightarrow (r_{11} + \imath d_d)/(1 + \imath d_d), \quad r_{12} \rightarrow (r_{12} - \imath d_d)/(1 + \imath d_d), \quad \text{and} \quad h \rightarrow h(1 + \imath d_d).$$

But, no generalization of reduced cases can yield the propagation regimes of anisotropic permeability.

5. Numerical computation and discussion

The analytical expressions derived in previous sections represent the most general mathematical model for wave propagation in a saturated poroelastic solid. These expressions can be used to compute the effects of (a) elastic anisotropy (different symmetries) of poroelastic solid, (b) viscosity of pore fluid, (c) hydraulic (permeability) anisotropy of different symmetries, (d) size and shape of pores, (e) porosity of saturated porous solid, and (f) wave frequency on the phase propagation velocities and attenuation of quasi-waves in the medium. The present numerical work is, however, restricted to study the effects of viscosity and hydraulic anisotropy. Write the dissipation matrix $\mathbf{d} = \imath\Delta_0\chi_a^{-1}$, where χ_a is normalized permeability tensor. A dimensionless parameter $\Delta_0 = \frac{f\rho_f}{\rho_{22}} / \frac{\omega}{\omega_c} = (1 + r_{12})\frac{\omega_c}{\omega}$ is defined that decides the frequency regime of Biot's theory through the characteristic frequency (ω_c). The Poiseuille flow ($\omega \ll \omega_c$) in pores is ensured by the value of $\omega/\omega_c < 0.1$. This restricts the value of $\Delta_0 > 10(1 + r_{12})$ for low-frequency wave propagation regime of Biot's theory. Similarly, high-frequency regime of $\omega/\omega_c > 10$ restricts the value of $\Delta_0 < 0.1\ (1 + r_{12})$.

Analysis of phase velocity, and attenuation in a real crystal may be a useful study. Elastic matrix (GPa) for dolomite, an anisotropic reservoir rock, following Rasolofosaon and Zinszner (2002), is written as

$$c_{11} = 65.53, \quad c_{12} = 9.77, \quad c_{13} = 12.19, \quad c_{14} = 0.18Z_1, \quad c_{15} = -0.81Z_1, \quad c_{16} = 2.94Z_2,$$

$$c_{22} = 50.77, \quad c_{23} = 11.61, \quad c_{24} = -0.09Z_1, \quad c_{25} = -0.50Z_1, \quad c_{26} = -0.19Z_2,$$

$$c_{33} = 60.11, \quad c_{34} = -1.61Z_1, \quad c_{35} = 1.78Z_1, \quad c_{36} = 0.84Z_2,$$

$$c_{44} = 23.51, \quad c_{45} = 1.49Z_2, \quad c_{46} = -1.17Z_1, \quad c_{55} = 24.57, \quad c_{56} = 0.26Z_1, \quad c_{66} = 20.21.$$

The values (GPa) of $\{m_{11}, m_{22}, m_{33}, m_{12}, m_{13}, m_{23}\} = \{6.5, 6, 5.5, 0.6Z_2, 0.7Z1, 0.5Z_1\}$ and $R = 2$ are assumed to represent a general anisotropic elastic coupling between fluid and solid constituents of porous aggregate. The above elastic matrices with value of $Z_1 = Z_2 = 1$ define the triclinic system of anisotropy. The values $Z_1 = 0, Z_2 = 1$ represent the monoclinic symmetry and the values $Z_1 = Z_2 = 0$ represent the orthorhombic symmetry. Dynamical constants are derived for 23% porosity, in a solid of density 2.423 g/cc and containing a fluid of density 0.98 g/cc. These are found to be, approximately, $\rho_{11} = 1.77$ g/cc; $\rho_{12} = -0.01$ g/cc; $\rho_{22} = 0.235$ g/cc. The symmetric tensor of anisotropic permeability for dolomite is given

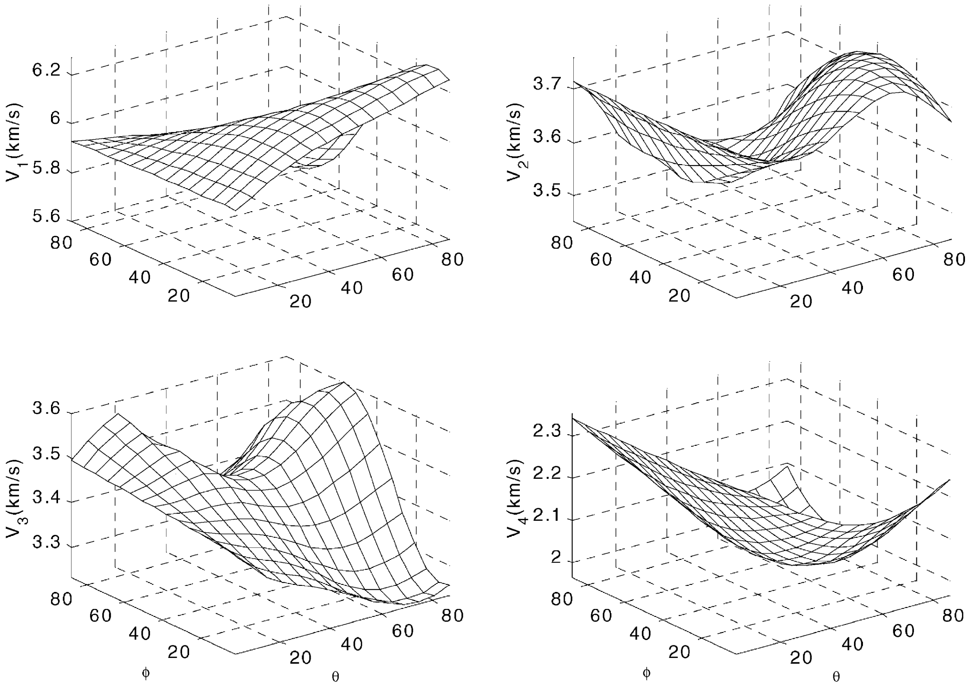

Fig. 1. Variations of phase propagation velocities (V_j) with the phase direction (θ, ϕ), in non-dissipative dolomite (anisotropic poroelastic medium); all angles are in degrees.

by, $\chi_a = \{0.96, -0.08, -0.06; -0.08, 0.72, 0.01; -0.06, 0.01, 0.73\}$. Identity matrix for χ_a represents the isotropic permeability. For the anisotropies with symmetries, the matrix χ_a can be defined similar to the matrix of $\{m_{ij}\}$. Using the above numerical values, the variations of phase propagation velocities (V_j), attenuation quality factors (Q_j^{-1}) with the phase direction are computed. The phase direction (θ, ϕ) varies from $(0, 0)$ to (90°, 90°).

Fig. 1 exhibits the variations of velocities with the phase direction in the anisotropic poroelastic (APE) solid saturated with a non-viscous fluid (i.e., $\Delta_0 = 0$). These plots serve as a platform to study the effects of fluid viscosity and solid permeability on wave propagation. It may be noted that attenuation is not there in such a medium.

The propagation velocities (V_j) are plotted in Figs. 2 and 3, for the different frequency regimes of Biot's theory. Medium of propagation, here, is anisotropic porous solid saturated by a viscous fluid. Poiseuille flow in pores is ensured by the value of $\Delta_0 = 20$. This represents the low-frequency regime (LF) of Biot's theory. The high-frequency regime (HF) is represented by the value of $\Delta_0 = 0.05$. Fig. 2 exhibits the velocity variations for isotropic permeability whereas, permeability with general anisotropy is considered for plots in Fig. 3. Comparison of these figures with Fig. 1 measures the effect of viscosity on velocities of propagation. The column-wise comparison of plots in these figures measure the effect of frequency regimes. The second and third columns of plots in these figures differs on the value of κ, which represents the viscosity of the interstitial fluid and size of pores. It is observed that presence of viscosity in the pore fluid slows down the propagation. Also note that slower the wave larger the effect. This effect of viscosity disappears a lot in high-frequency regime. As compared to viscosity, the effect of hydraulic anisotropy is very small in the all frequency regimes.

The attenuation variations of quasi-waves corresponding to the velocities in Figs. 2 and 3 are exhibited in Figs. 4 and 5, respectively. It may be noted that attenuation is largest for qS2-waves and negligible for

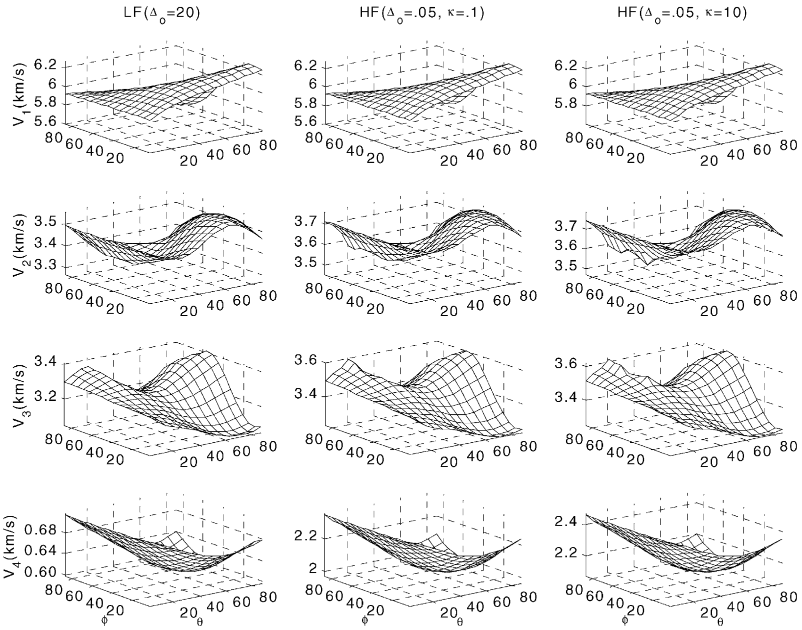

Fig. 2. Variations of phase velocities (V_j) of quasi-waves with the phase direction (θ, ϕ), in dissipative dolomite with isotropic permeability; all angles are in degrees.

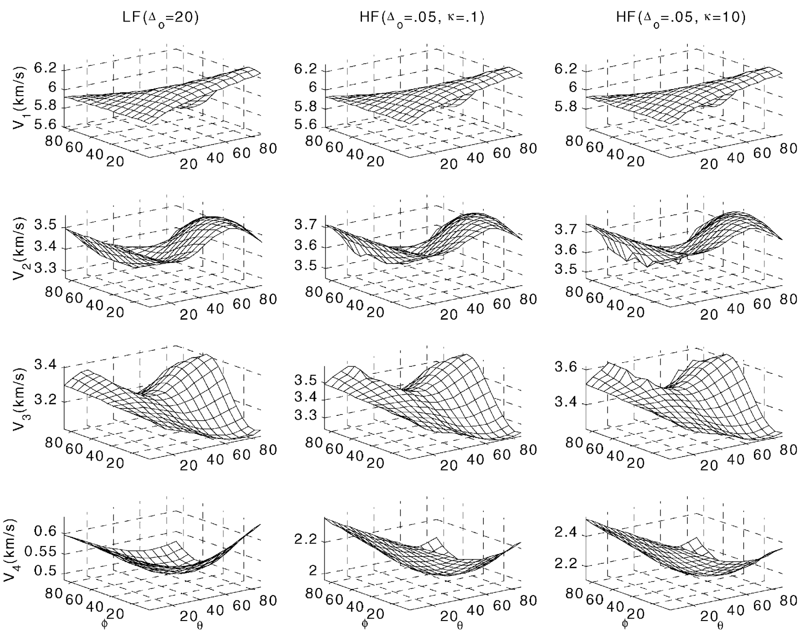

Fig. 3. Variations of phase velocities (V_j) of quasi-waves with the phase direction (θ, ϕ), in dissipative dolomite with anisotropic permeability; all angles are in degrees.

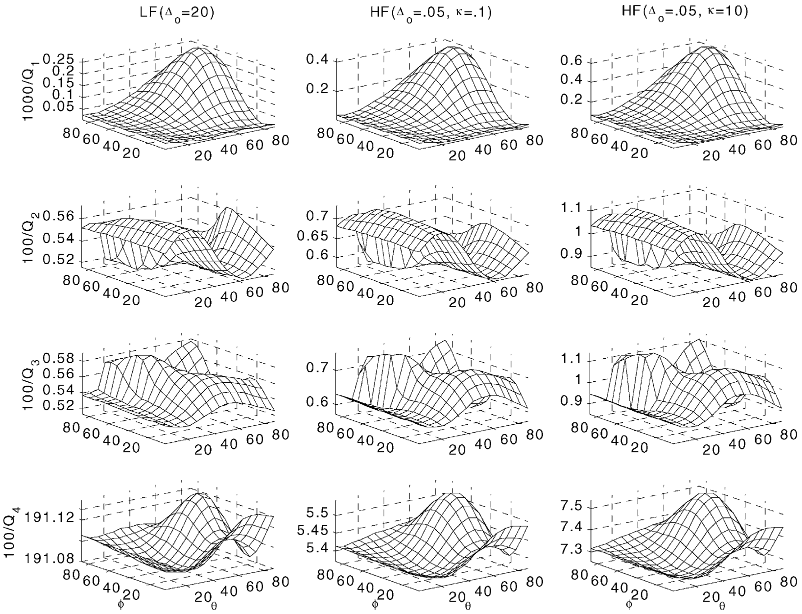

Fig. 4. Variations of quality factors (Q_j) of quasi-waves with the phase direction (θ, ϕ), in dissipative dolomite with isotropic permeability; all angles are in degrees.

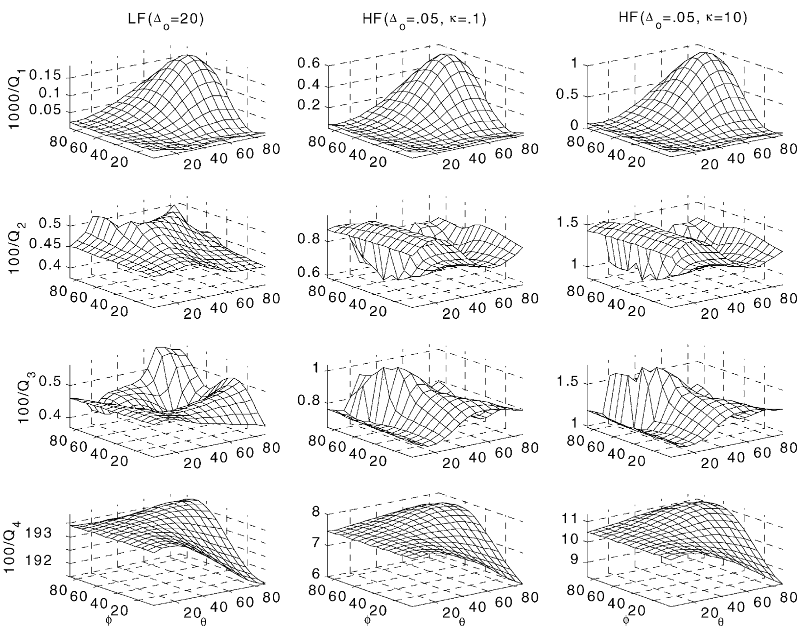

Fig. 5. Variations of quality factors (Q_j) of quasi-waves with the phase direction (θ, ϕ), in dissipative dolomite with anisotropic permeability; all angles are in degrees.

qP1-waves. On the increase of frequency to abandon Poiseuille flow, the attenuation of qS2-waves reduces drastically whereas it increases slightly for the remaining three quasi-waves. Increase of κ also increases the attenuation. In the high-frequency regime, the presence of hydraulic anisotropy shows a significant increase in the attenuations.

From the numerical results it is observed that anisotropic wave propagation is possible even in the poroelastic medium with isotropic solid matrix. It is possible, in high-frequency regime, when solid is saturated by viscous fluid with anisotropic permeability controlling its flow. Four quasi-waves do propagate in such a medium but the directional variations of phase velocities are very small. For low-frequency propagation (i.e., Poiseuille flow in pores), such an anisotropy is just negligible.

6. Conclusions

The above discussion of numerical results is made for a particular model. It may not qualify for generalization but conclusions drawn from it can certainly help in improving the mathematical models of wave propagation in APE solids. Discussion in the previous section may be interpreted for the following conclusions:

(a) Variations of phase propagation velocities with phase direction
1. Presence of viscosity in the pore fluid reduces the velocities of all the quasi-waves to a large extent (compare Figs. 2 and 3 with 1). Waves propagating with velocity V_4 slows down nearly four times. In general, faster the wave smaller the effect.
2. In the low-frequency regime (Poiseuille flow) hydraulic anisotropy have a very little decreasing effect on the V_4 and almost no effect on other three quasi-waves.
3. The frequency increase ($\kappa = 0.1$ case) do increase the velocities of all the waves to their corresponding velocities of non-dissipative case (Fig. 1). This implies that the effect of viscosity is nullified by the smaller size of pores which denies the motion of pore fluid relative to the solid. The presence of hydraulic anisotropy, in this case, slows down the waves a little bit.
4. The increase of κ ($\kappa = 10$ case, i.e., wider pores and/or smaller viscosity) increases the velocities of slower waves. In this case hydraulic anisotropy also increases these velocities but only by small amount of 2–5%.
5. The hydraulic anisotropy may lead to anisotropic propagation in an isotropic porous solid. Such an anisotropic propagation is small and restricted to high-frequency regime.

(b) Variations of attenuation with phase direction
6. Presence of viscosity in the pore fluid introduces the attenuation in the amplitudes of the quasi waves (Fig. 3). Attenuation is much large for the quasi-wave propagating at the slowest velocity V_4.
7. In the low-frequency regime (i.e., Poiseuille flow) hydraulic anisotropy reduces the attenuation of faster waves to some extent (Figs. 4 and 5). Attenuation of fastest wave is negligible.
8. In the high-frequency regime the attenuations of three of the four quasi-waves, are larger to that in low-frequency regime. The attenuation of slowest wave (i.e., $100/Q_4$), however, decreases up to 30 times in high-frequency. Hydraulic anisotropy do increase the attenuation considerably.
9. Decrease of viscosity and/or increase of pore size in the high-frequency regime (i.e., change of κ from 0.1 to 10) increase the attenuation of all the quasi-waves. Increase in attenuation due to the hydraulic anisotropy is much larger as compared to the effect on velocities.

This piece of work studies the wave propagation in a realistic medium keeping in mind the physical properties of reservoir rocks. The theoretical and calculation schemes are used to their utmost capabilities to derive a most general mathematical model for wave propagation in a general anisotropic poroelastic solid. The prospecting seismologists and researchers in this field would prefer to use this model, for the interpretation of their complex data. The work presented can, further, be used to study the polarizations, surface waves and scattering in a general anisotropic poroelastic medium. The study of anisotropic poroelasticity may also be important for understanding the mechanical behaviour of composite materials leading to enormous applications in non-destructive testing/evaluation studies (Braga, 1990; Buden and Datta, 1990; Chai and Wu, 1994; Wu and Wu, 2000).

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